Simplify Each

1) $\cot^2 x - \csc^2 x$

Use Pythagorean Identity

$\csc^2 x - \cot^2 x = 1$

So if I multiply by -1

$-1(\csc^2 x - \cot x) = -1(1)$

$-\csc^2 x + \cot^2 x = -1$

re-order

$\cot^2 x - \csc^2 x = -1$

Same as original

Answer is $\boxed{-1}$

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2) $(1 - \cos^2 x) \cdot (\csc x)$

Pythagorean identity

$(\sin^2 x) \cdot (\csc x)$

Convert to $\frac{1}{\sin}$

$(\sin^2 x) \cdot \frac{1}{\sin x} \rightarrow \frac{\sin^2 x}{\sin x}$ Reduce $\rightarrow \boxed{\sin x}$

(3) $\sec^2 x \left(1 - \sin^2 x\right)$

↑ Pythagorean Identity

$\sec^2 x \left(\cos^2 x\right)$

↑ Convert to cos

$\dfrac{1}{\cos^2 x} \cdot \left(\cos^2 x\right)$

$\dfrac{\cos^2 x}{\cos^2 x} = \boxed{1}$

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(4) $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$ ⇄ These are both Pythagorean Identities

$\dfrac{\cos^2 x}{\cot^2 x}$ ← Now convert $\cot^2 x$ to sin/cos

$\dfrac{\cos^2 x}{\dfrac{\cos^2 x}{\sin^2 x}}$ Then keep / change / flip

$\cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x}$ Reduce

$\dfrac{\cos^2 x \, \sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$

⑤ $\dfrac{1}{\tan^2 x + 1}$ ← This is a Pythagorean Identity

$\dfrac{1}{\sec^2 x}$ then keep/change flip

$1 \cdot \dfrac{\sec^2 x}{1} = \boxed{\sec^2 x}$

⑥ $\tan^2 x - \tan^2 x \sin^2 x$ GCF first!

$\tan^2 x \left(1 - \sin^2 x \right)$

↖ This is a Pythagorean Identity

$\tan^2 x \left(\cos^2 x \right)$

↖ Convert to sin/cos

$\dfrac{\sin^2 x}{\cos^2 x} \left(\cos^2 x \right)$ Reduce $\dfrac{\sin^2 x}{\cos^2 x} \left(\cos^2 x \right) = \boxed{\sin^2 x}$

⑦ $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$ Get a Common Denominator

$\dfrac{1(1 - \cos x)}{(1 + \cos x)(1 - \cos x)} + \dfrac{1(1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$ Distribute + write as 1 fraction

$\dfrac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$ Combine like terms in numerator

$\dfrac{2}{(1 + \cos x)(1 - \cos x)}$ Multiply (FOIL) denominator $\rightarrow \dfrac{2}{1 - \cos^2 x} \rightarrow \boxed{\dfrac{2}{\sin^2 x}}$

← Pythagorean identity