(1) $\int x^2(2-3x^3)^4\,dx$

let $u = 2-3x^3$ $\quad du = -9x^2\,dx$

$dx = \frac{du}{-9x^2}$

$\int x^2 u^4 \frac{du}{-9x^2}$

$\frac{-1}{9}\int u^4\,du$

$\frac{-1}{9}\frac{u^5}{5}$

$\frac{-1}{45}u^5$

$-\frac{1}{45}(2-3x^3)^5 + C$

(2) $\int \frac{3x}{\sqrt[3]{3-7x^2}}\,dx$

let $u = 3-7x^2$ $\quad du = -14x\,dx$

$dx = \frac{du}{-14x}$

$\int \frac{3x}{\sqrt[3]{u}} \frac{du}{-14x}$

$-\frac{3}{14}\int \frac{1}{\sqrt[3]{u}}\,du$

$-\frac{3}{14}\int \frac{1}{u^{1/3}}\,du$

$-\frac{3}{14}\int u^{-1/3}\,du$

$-\frac{3}{14}\frac{u^{2/3}}{2/3}$

$-\frac{3}{14}\cdot\frac{2}{3}(u)^{2/3}$

$-\frac{1}{7}(3-7x^2)^{2/3} + C$

(3) $\int (3x-1)^5\,dx$

let $u = 3x-1$ $\quad du = 3\,dx$

$dx = \frac{du}{3}$

$\frac{1}{3}\int u^5\,du$

$\frac{1}{3}\frac{u^6}{6}$

$\frac{1}{18}u^6$

$\frac{1}{18}(3x-1)^6 + C$

(4) $\int x(2-x^2)^3\,dx$

let $u = 2-x^2$ $\quad du = -2x\,dx$

$dx = \frac{du}{-2x}$

$\int x u^3 \frac{du}{-2x}$

$-\frac{1}{2}\int u^3\,du$

$-\frac{1}{2}\frac{u^4}{4}$

$\frac{1}{2}\frac{(2-x^2)^4}{4}$

$\frac{1}{8}(2-x^2)^4 + C$

(5) $\int (x^2+1)(x^3+3x)^{10}\,dx$

let $u = x^3+3x$ $\quad du = 3x^2+3\,dx$

$dx = \frac{du}{3x^2+3}$

$\int (x^2+1)(u)^{10}\frac{du}{3x^2+3}$

$\int (\cancel{x^2+1})(u)^{10}\frac{du}{(3)(\cancel{x^2+1})}$

$\frac{1}{3}\int u^{10}\,du$

$\frac{1}{3}\ \frac{u^{11}}{11}$

$\frac{1}{33}(x^3+3x)^{11}+C$

(6) $\int \sin(3x)\,dx$

let $u = 3x$ $\quad du = 3\,dx$

$dx = \frac{du}{3}$

$\int \sin(u)\frac{du}{3}$

$\frac{1}{3}\int \sin u\,du$

$\frac{1}{3}(-\cos u)$

$-\frac{1}{3}\cos u$

$-\frac{1}{3}\cos(3x)+C$

(7) $\int 4x(5-3x^2)^3\,dx$

let $u = 5-3x^2$ $\quad du = -6x\,dx$

$dx = \frac{du}{-6x}$

$\int 4x(u^3)\frac{du}{-6x}$

$-\frac{4}{6}\int u^3\,du$

$-\frac{2}{3}\int u^3\,du$

$-\frac{2}{3}\ \frac{u^4}{4}$

$-\frac{2}{12}u^4$

$-\frac{1}{6}u^4$

$-\frac{1}{6}(5-3x^2)^4+C$

(8) $\int \frac{x^2}{\sqrt{x^3+1}}\,dx$

let $u = x^3+1$ $\quad du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int \frac{\cancel{x^2}}{\sqrt{u}}\ \frac{du}{3\cancel{x^2}}$

$\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

$\frac{1}{3}\int \frac{1}{u^{1/2}}\,du$

$\frac{1}{3}\int u^{-1/2}\,du$

$\frac{1}{3}\ \frac{u^{1/2}}{\frac{1}{2}}$ $\rightarrow$ $\frac{1}{3}\cdot\frac{2}{1}u^{1/2}$

$\frac{2}{3}(x^3+1)^{1/2}+C$