(1) $y = x^3 + x \quad [0,3]$

$b \quad (h_1 + h_2 + h_3 \cdots h_n)$

base: $\frac{3-0}{n}$

$$\frac{3}{n}\left(f\left(\frac{3}{n}\right) + f\left(\frac{6}{n}\right) + f\left(\frac{9}{n}\right) \cdots f\left(\frac{3n}{n}\right)\right)$$

$$\frac{3}{n}\left(\left(\frac{3}{n}\right)^3 + \left(\frac{3}{n}\right) + \left(\frac{6}{n}\right)^3 + \left(\frac{6}{n}\right) + \left(\frac{9}{n}\right)^3 + \left(\frac{9}{n}\right) \cdots \left(\frac{3n}{n}\right)^3 + \left(\frac{3n}{n}\right)\right)$$

$$\frac{3}{n}\left(\frac{27}{n^3} + \frac{3}{n} + \frac{216}{n^3} + \frac{6}{n} + \frac{729}{n^3} + \frac{9}{n} \cdots \frac{27n^3}{n^3} + \frac{3n}{n}\right)$$

$$\frac{3}{n}\left(\frac{27}{n^3} + \frac{216}{n^3} + \frac{729}{n^3} + \frac{27n^3}{n^3} + \frac{3}{n} + \frac{6}{n} + \frac{9}{n} \cdots \frac{3n}{n}\right)$$

$$\frac{3}{n}\left[\frac{27}{n^3}\left(1 + 8 + 27 \cdots n^3\right) + \frac{3}{n}\left(1 + 2 + 3 \cdots n\right)\right]$$

$$\frac{81}{n^4}\left(k^3\right) + \frac{9}{n^2}\left(k\right)$$

$$\frac{81}{n^4}\left(\frac{n^4 + 2n^3 + n^2}{4}\right) + \frac{9}{n^2}\left(\frac{n^2 + n}{2}\right)$$

$$\frac{81n^4}{4n^4} + \frac{162n^3}{4n^4} + \frac{81n^2}{4n^4} + \frac{9n^2}{2n^2} + \frac{9n}{2n^2}$$

$$\frac{81}{4} + \frac{9}{2}$$

$99/4$

(2) $y = x^2 + 4x \quad [0,1]$

Base $\frac{1-0}{n} = \frac{1}{n}$

$b(h_1 + h_2 + h_3 + \dots h_n)$

$\frac{1}{n}\left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + f\left(\frac{3}{n}\right) \dots f\left(\frac{1n}{n}\right)\right)$

$\frac{1}{n}\left(\left(\frac{1}{n}\right)^2 + 4\left(\frac{1}{n}\right) + \left(\frac{2}{n}\right)^2 + 4\left(\frac{2}{n}\right) + \left(\frac{3}{n}\right)^2 + 4\left(\frac{3}{n}\right) \dots \left(\frac{n}{n}\right)^2 + 4\right.$

$\frac{1}{n}\left(\frac{1}{n^2} + \frac{4}{n} + \frac{4}{n^2} + \frac{8}{n} + \frac{9}{n^2} + \frac{12}{n} \dots \frac{n^2}{n^2} + \frac{4n}{n} \right)$

$\frac{1}{n}\left(\frac{1}{n^2} + \frac{4}{n^2} + \frac{9}{n^2} \dots \frac{n^2}{n^2} + \frac{4}{n} + \frac{8}{n} + \frac{12}{n} \dots \frac{4n}{n} \right).$

$\frac{1}{n}\left(\frac{1}{n^2}\left(1 + 4 + 9 + \dots n^2\right) + \frac{4}{n}\left(1 + 2 + 3 \dots n\right)\right)$

$\frac{1}{n^3}\left(k^2\right) + \frac{4}{n^2}\left(k\right)$

$\frac{1}{n^3}\left(\frac{2n^3 + 3n^2 + n}{6}\right) + \frac{4}{n^2}\left(\frac{n^2 + n}{2}\right)$

$\frac{2n^3}{6n^3} + \frac{3n^2}{6n^3} + \frac{1n}{6n^3} + \frac{4n^2}{2n^2} + \frac{4n}{2n^2}$

$\frac{2}{6} + 2$

$\frac{1}{3} + \frac{6}{3} = \boxed{7/3}$

(3) $y = 3x^2 \quad [0,2]$

$b(h_1 + h_2 + h_3 \ldots\ldots h_n)$

base $\frac{2-0}{n}$

$$\frac{2}{n}\left(f\left(\frac{2}{n}\right) + f\left(\frac{4}{n}\right) + f\left(\frac{6}{n}\right) \ldots\ldots f\left(\frac{2n}{n}\right)\right)$$

$$\frac{2}{n}\left(3\left(\frac{2}{n}\right)^2 + 3\left(\frac{4}{n}\right)^2 + 3\left(\frac{6}{n}\right)^2 \ldots\ldots 3\left(\frac{2n}{n}\right)^2\right)$$

$$\frac{2}{n}\left(\frac{12}{n^2} + \frac{48}{n^2} + \frac{108}{n^2} \ldots\ldots \frac{12n^2}{n^2}\right)$$

$$\frac{2}{n} \cdot \frac{12}{n^2}\left(1 + 4 + 9 \ldots\ldots n^2\right)$$

$$\frac{24}{n^3}\left(K^2\right)$$

$$\frac{24}{n^3}\left(\frac{2n^3 + 3n^2 + n}{6}\right)$$

$$\frac{48n^3}{6n^3} + \frac{72n^2}{6n^3} + \frac{24n}{6n^3}$$

$$48/6 = 8$$

4)

$$\sum_{k=41}^{53} k^3 + 2k^2$$

$$\sum_{k=1}^{53} \left[\frac{n(n+1)}{2}\right]^2 + 2\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\left[\frac{53(54)}{2}\right]^2 + 2\left(\frac{53(54)(107)}{6}\right)$$

$$2047761 + 102078 \qquad 2149839$$

$$\sum_{k=1}^{40} \left[\frac{n(n+1)}{2}\right]^2 + 2\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\left[\frac{40(41)}{2}\right]^2 + 2\left(\frac{40(41)(81)}{6}\right)$$

$$672400 + 44280 \qquad - 716680$$

$$1,433,159$$