① $y = (x-4)^{2/3}$

$y' = \frac{2}{3}(x-4)^{-1/3}$ Save for 2nd derivative

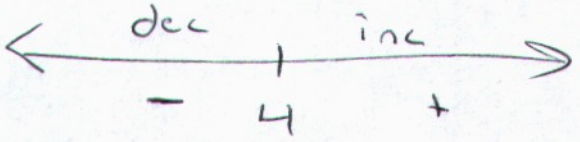
$y' = \frac{2}{3(x-4)^{1/3}}$

$\frac{2}{3(x-4)} = \frac{0}{1}$

$2 \neq 0$ no answer

POND @ $x = 4$

f' test



~~POND~~ (4, 0)

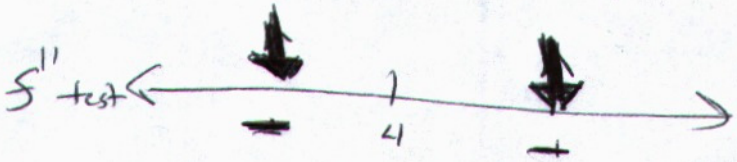
$y'' = -\frac{2}{9}(x-4)^{-4/3}$

$y'' = \frac{-2}{9(x-4)^{4/3}} = \frac{0}{1}$

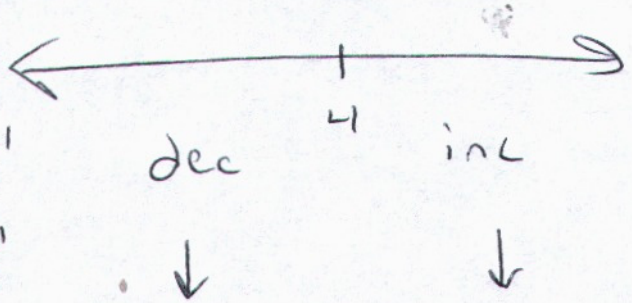
$-2 \neq 0$ no answer

POND @ $x = 4$

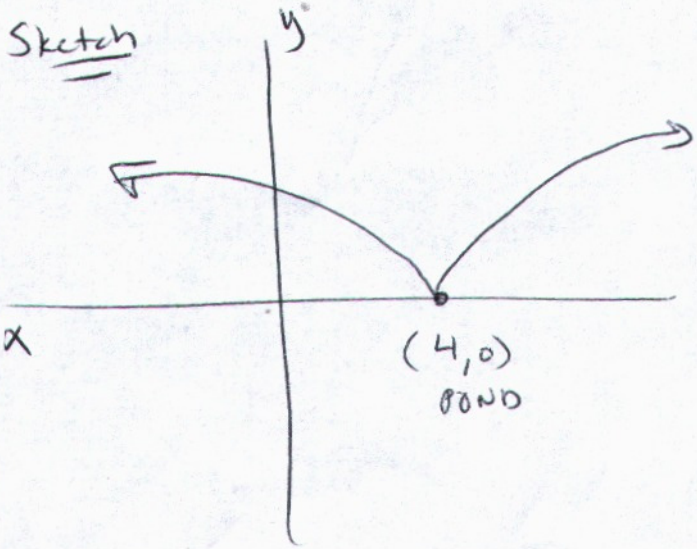
f'' test



BIC



Sketch



(2)

$f(x) = 3x^{1/3} - 2x$

$f'(x) = 1x^{-2/3} - 2$ Save for 2nd derivative

$f'(x) = \frac{1}{x^{2/3}} - 2$ Common denominator

$f'(x) = \frac{1}{x^{2/3}} - \frac{2x^{2/3}}{x^{2/3}}$

$f'(x) = \frac{1 - 2x^{2/3}}{x^{2/3}} = \frac{0}{1}$

$1 - 2x^{2/3} = 0$

$-2x^{2/3} = -1$

$x^{2/3} = 1/2$

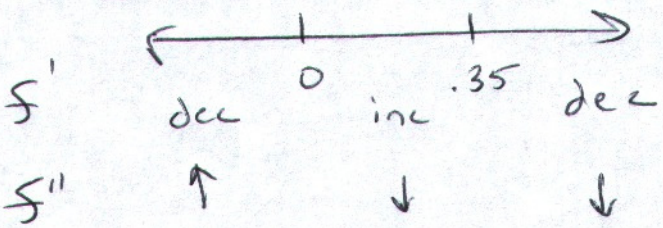
$(x^{2/3})^{3/2} = (1/2)^{3/2}$

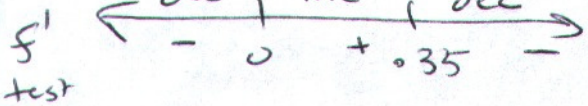
$x \approx .35$

POND @ $x = 0$

POND $(0, 0)$

B I C





TP $(.35, 1.24)$

POND $(0, 0)$

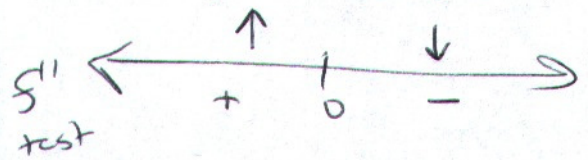
$f'' = 1x^{-2/3} - 2$

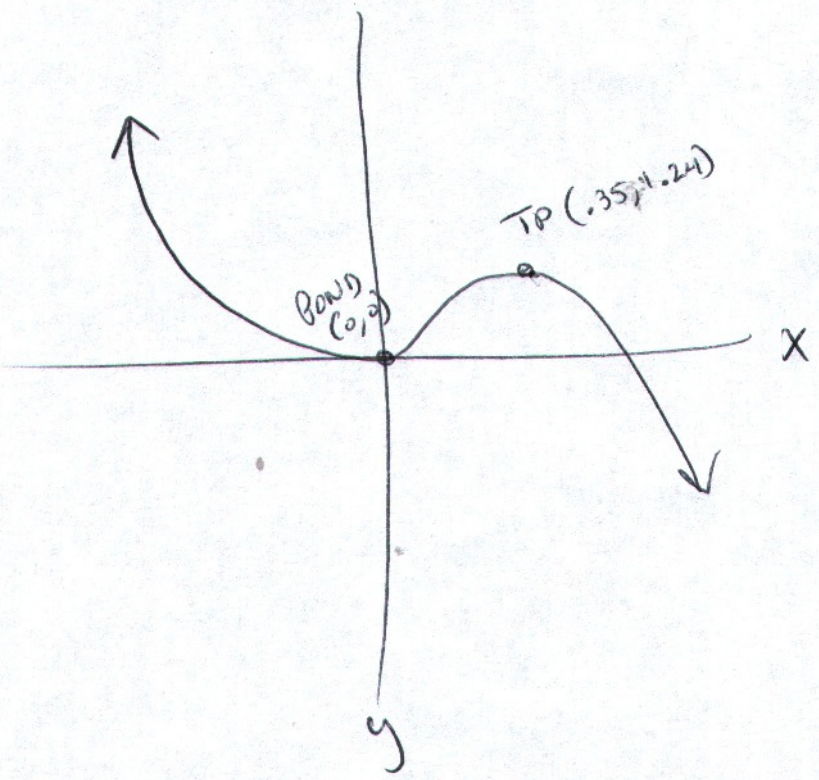
$f'' = -\frac{2}{3}x^{-5/3}$

$f'' = \frac{-2}{3x^{5/3}} = \frac{0}{1}$

$-2 \neq 0$ no answer

POND @ $x = 0$





(3) $f(x) = 7x - 3x^{2/3}$

$f' = 7 - 2x^{-1/3}$ Save for 2nd derivative

$f' = 7 - \frac{2}{x^{1/3}}$

Common denominator

$f' = \frac{7x^{1/3}}{x^{1/3}} - \frac{2}{x^{1/3}}$

$f' = \frac{7x^{1/3} - 2}{x^{1/3}} = \frac{0}{1}$

$7x^{1/3} - 2 = 0$

$7x^{1/3} = 2$

$x^{1/3} = \frac{2}{7}$

$(x^{1/3})^{3/1} = (\frac{2}{7})^{3/1}$

$x \approx .02$ Pond ⓐ $x=0$

inc | dec | inc

\+ 0 − .02 +

POND (0,0) TP (.02, −.08)

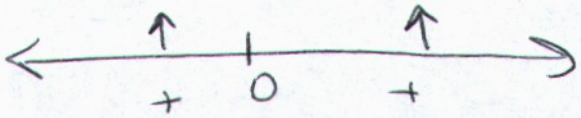
$f'' = 7 - 2x^{-1/3}$

$f'' = \frac{2}{9}x^{-4/3}$

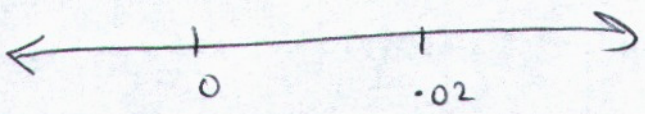
$f'' = \frac{2}{9x^{4/3}} = \frac{0}{1}$

$2 \neq 0$ no answer

Pond ⓐ $x=0$

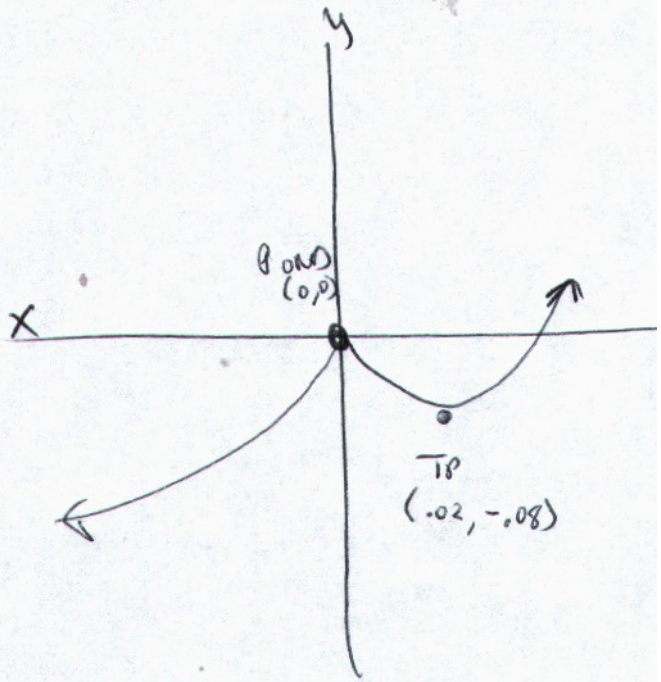


BIC



f' inc dec inc

f'' ↑ ↑ ↑



(4) $y = \dfrac{3x}{x+4}$ ← original is a fraction check for asymptotes

Vertical Asymptote $x = -4$

Horizontal Asymptote $y = 3$

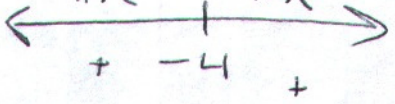
$$y' = \frac{(x+4)(3) - (3x)(1)}{(x+4)^2}$$

$$y' = \frac{12}{(x+4)^2} = \frac{0}{1}$$

$12 \neq 0$ no answer

POND @ $x = -4$

f' test



$$y'' = \frac{(x+4)^2(0) - (12)(2(x+4)'(1))}{(x+4)^4}$$ ← chain rule

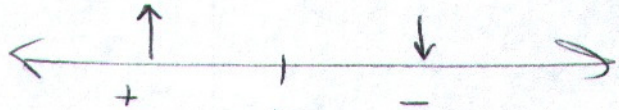
$$y'' = \frac{-24x - 96}{(x+4)^4} = \frac{0}{1}$$

$-24x - 96 = 0$

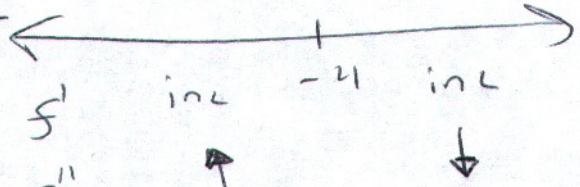
$\dfrac{-24x}{-24} = \dfrac{96}{-24}$

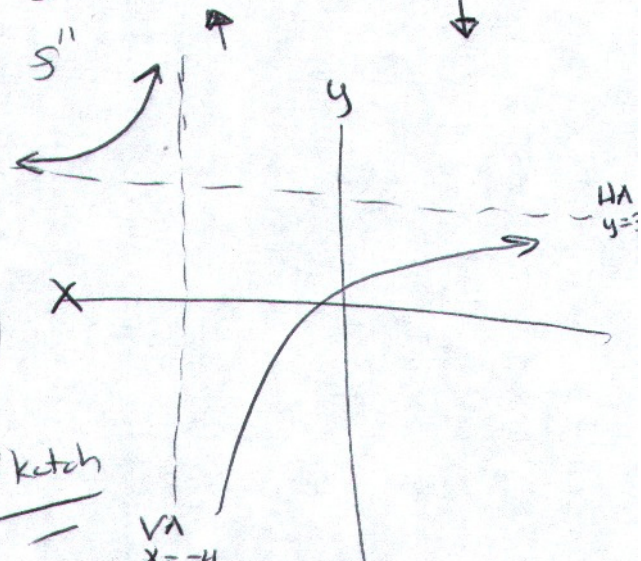
$x = -4$

f'' test



BTC





Sketch

(5) $f(x) = \dfrac{5x^2}{x^2-9}$ ← original is a fraction check for asymptotes

VA $x = 3$ $x = -3$

HA $y = 5$

$$f'(x) = \frac{(x^2-9)(10x) - (5x^2)(2x)}{(x^2-9)^2}$$

$$f'(x) = \frac{\cancel{10x^3} - 90x - \cancel{10x^3}}{(x^2-9)^2}$$

$$f' = \frac{-90x}{(x^2-9)^2} = \frac{0}{1}$$

$-90x = 0$

$x = 0$ POND $x = \pm 3$

inc | dec | dec | dec

f' test -3 + 0 − 3 −

TP $(0, 0)$

POND $(-3,$ undefined$)$

$(3,$ undefined$)$

$$f' = \frac{-90x}{(x^2-9)^2}$$

All have GCF of $(x^2-9)^1$

$$f'' = \frac{(x^2-9)^2(-90) - (-90x)(2(x^2-9)^1)(2x)}{(x^2-9)^4}$$

chain rule

$$f'' = \frac{(x^2-9)(-90) + 360x^2}{(x^2-9)^3}$$

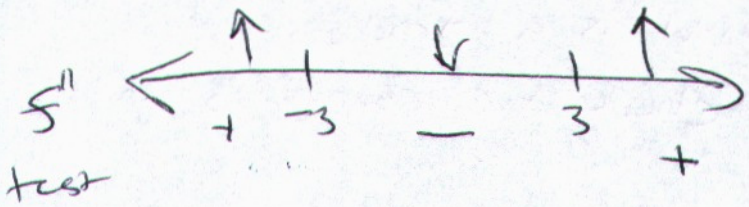
$$f'' = \frac{-90x^2 + 810 + 360x^2}{(x^2-9)^3} = \frac{0}{1}$$

$270x^2 + 810 = 0$

$x^2 = -\frac{810}{270}$

$x = \sqrt{-3}$ imaginary

but POND (9) $x = \pm 3$



B I C