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Name Key

**Unit 1: Sequences**

**Review Sheet**

1. Express the general term of this sequence in bracket notation

$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots \qquad \left\{\frac{n}{2n-1}\right\}_{n=1}^{\infty}$

2. Does the sequence below converge or diverge? If it converges, find its limit

$\left\{n^{2/n}\right\}_{n=1}^{\infty} \qquad \lim_{n\to\infty} ? \qquad 1^2, 2^{1/2}, 3^{2/3}, 4^{2/4}, 5^{2/5}$

3. Does the given sequence converge or diverge?

$\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty} \qquad \lim_{n\to\infty} = \frac{1}{2}$ Converges

4. Give general term of this sequence. Determine if it converges or diverges. If converges, give the limit

$\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \ldots \qquad \left\{\frac{n^2}{n+1}\right\}_{n=1}^{\infty} \qquad \lim_{n\to\infty} = \infty$

Diverges

5. Use the DIFFERENCE of successive terms to determine if the sequence is monotone. If so, classify (inc/dec nondec/noninc)

$\left\{1 - \frac{2}{n}\right\}_{n=1}^{\infty}$

$-n^2 + n + 2$

$n^2 - 2n + n - 2$

$\left(1 - \frac{2}{n+1}\right) - \left(1 - \frac{2}{n}\right) \qquad \frac{(n)\,n-1}{(n)\,n+1} - \frac{n-2\,(n+1)}{n\,(n+1)}$

$\left(\frac{n+1-2}{n+1}\right) - \left(\frac{n-2}{n}\right) \qquad \frac{n^2 - n \cancel{-n^2} \cancel{+n} + 2}{n(n+1)} \qquad \frac{2}{n(n+1)} > 0$ for all $n$

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**Unit 1: Sequences**

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6. Use the DIFFERENCE of successive terms to determine if the sequence is monotone. If so, classify (inc/dec nondec/noninc)

$$\left\{ \frac{2n}{2n-1} \right\}_{n=1}^{\infty} \qquad \frac{2(n+1)}{2(n+1)-1} - \frac{2n}{2n-1} \rightarrow \frac{(2n+2)}{2n+1}(2n-1) + \frac{-2n}{2n-1}(2n+1)$$

$$\frac{2n+2}{2n+1} \qquad \frac{4n^2 \cancel{-2n} + 4n - 2 \cancel{-4n^2} \cancel{-2n}}{CD} \qquad \frac{-2}{(2n+1)(2n-1)}$$

Decreasing $<0$ for all "$n$"

7. Use the RATIO of successive terms to determine if the sequence is monotone. If so, classify (inc/dec nondec/noninc)

$$\left\{ \frac{3^n}{n!} \right\}_{n=1}^{\infty} \qquad \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} \qquad \frac{\cancel{3^n} \cdot 3^1}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{3^n}} \qquad \frac{3}{n+1}$$

→ Look at terms

This is only sometimes $< 1$

Only decreasing for $n > 2$

8. Use the DERIVATIVE of successive terms to determine if the sequence is monotone. If so, classify (inc/dec nondec/noninc)

$$\left\{ \ln\left(\frac{2n}{n+1}\right) \right\}_{n=1}^{\infty} \qquad \frac{1}{\left(\frac{2n}{n+1}\right)} \cdot \frac{(n+1)(2) - (2n)(1)}{(n+1)^2}$$

$$\frac{1}{\frac{2n}{n+1}} \cdot \frac{2}{(n+1)^2}$$

$$\frac{\cancel{n+1}}{\cancel{2}n} \cdot \frac{\cancel{2}}{(n+1)^{\cancel{2}}} \rightarrow \frac{1}{n(n+1)}$$

This derivative will always be + for $n \geq 1$ so Monotone increasing

9. Use partial sums (partial fractions and telescoping) to determine if the sequence converges or diverges

$$\sum_{K=1}^{\infty} \frac{1}{K^2+5K+6}$$

$$\frac{1}{K^2+5K+6} = \frac{A}{K+3} + \frac{B}{K+2}$$

$$\frac{0K+1}{(K+3)(K+2)} = \frac{AK+2A+BK+3B}{(K+3)(K+2)}$$

So $0K = AK + BK$

or

$0 = A + B$

Solve $1 = 2A + 3B$

$A = -1$

$B = 1$

$$\frac{-1}{K+3} + \frac{1}{K+2}$$

$$\cancel{-\frac{1}{4}} + \frac{1}{3} \quad \cancel{-\frac{1}{5}} + \cancel{\frac{1}{4}} \quad \cancel{-\frac{1}{6}} + \cancel{\frac{1}{5}} \ldots$$

$1/3$