Name ___________________________________ AP Calculus BC Unit 9 Practice Test

Directions: Calculator use is permitted for questions 2, 4, 6, 7, 8, 10, 11, 14, and 15.

Part 1 – Multiple Choice (15 questions)

(1) For what value(s) of t does the curve given by the parametric equations $x = t^3 - 3t^2 - 2$ and $y = t^6 + 3t^2 - 12t$ have a vertical tangent?

(A) 0 only (B) 1 only (C) 0 and 2 only (D) 0, 1, and 2 only

(2) A curve is defined by parametric equations $x(t) = \frac{1}{3}t^3 - 4t$ and $y(t) = \frac{3}{2}t^2 - 5t$. For what value of t, $0 \leq t \leq 10, is\ \frac{d^2y}{dx^2} = 3$?

(A) 0.500

(B) 1.712

(C) 2.861

(D) There is no such value of t.

(3) A curve is defined by the parametric equations $x(t) = -3t$ and $y(t) = 2t^2 - 2t$. Which of the following is an equation of the line tangent to the graph of the curve at the point where t = 2?

(A) $y = -2x - 4$ (B) $y = -2x - 8$ (C) $y = -\frac{1}{2}x - 8$ (D) $y = -\frac{1}{2}x - 4$

(4) The position of a particle moving in the xy-plane at any time t is given by the vector-valued function $p(t) = < \frac{1}{3}t^3 + t,\ \frac{1}{4}t^4 + 4t >$. The magnitude of the velocity vector at $t = 2$ is

(A) 34.438

(B) 28.146

(C) 17

(D) 13

(5) If g is a vector-valued function defined by $g(t) = (\sin(3t), e^{-2t})$, then $g''(t) =$

(A) $(9\sin(3t), -4e^{-2t})$

(B) $(-9\sin(3t), 4e^{-2t})$

(C) $(\sin(3t), e^{-2t})$

(D) $(-\sin(3t), e^{-2t})$

(6) Consider the parametric curve C described by parametric equations x(t) and y(t), $t \geq 0$. If $x'(t) = t$ and $y'(t) = \sqrt{4t + 4}$, what is the length of curve C from t = 1 to t = 3?

(A) 2.518

(B) 4.651

(C) 8.000

(D) 32.667

(7) The velocity of a particle moving in the xy-plane is modeled by the vector-valued function $v(t) = < \frac{t-6}{2^{t-8}}, -\cos(3t) >, t \geq 0$. At what time t is the speed of the particle equal to 5?

(A) 3.812 (B) 5.264 (C) 8.036 (D) 10.757

(8) A particle moves in the xy plane in such a way that its acceleration can be modeled by the vector-valued function $a(t) = < 2t + 1, -2t + 3 >$. Let the vector-valued function v(t) represent the velocity of the particle where $v(1) = < 3, 5 >$. What is the total distance travelled by the particle from t = 0 to t = 2.

(A) 11.662

(B) 11.668

(C) 12.231

(D) 12.472

(9) A particle moves on a plane curve in such a way that its position can be modeled by the vector-valued function p(t), where $p(t) = < 3\sin t, 5\cos t >$. The acceleration vector of the particle at $t = \frac{\pi}{3}$ is

(A) $< -\frac{3\sqrt{3}}{2}, -\frac{5}{2} >$

(B) $< \frac{3\sqrt{3}}{2}, \frac{5}{2} >$

(C) $< -\frac{5}{2}, -\frac{3\sqrt{3}}{2} >$

(D) $< \frac{5}{2}, \frac{3\sqrt{3}}{2} >$

(10) A particle is moving along the polar function $f(\theta) = 2\sin\theta - 5\cos\theta$ so that at any time t seconds, $\theta = t$. The function $r = f(\theta)$ is measured in centimeters, and the particle is travelling along the curve from $t = 0\ to\ t = 2\pi$ seconds. Which of the following is best interpretation of $f'\left(\frac{\pi}{6}\right)$ in the context of the problem?

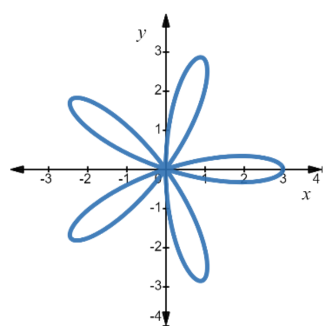
(A) The distance of the particle from the origin is increasing at a rate of 3.189 cm per second $t = \frac{\pi}{6}$.

(B) The distance of the particle from the origin is decreasing at a rate of 3.189 cm per second $t = \frac{\pi}{6}$.

(C) The distance of the particle from the origin is increasing 4.232 cm per second at time $t = \frac{\pi}{6}$.

(D) The distance of the particle from the origin is decreasing 4.232 cm per second at time $t = \frac{\pi}{6}$.

(11)

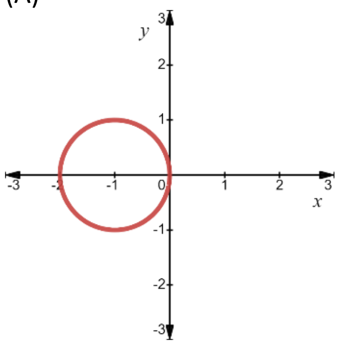


What is the area of one petal of the polar curve $r = 3\cos(5\theta)$ shown in the figure above?

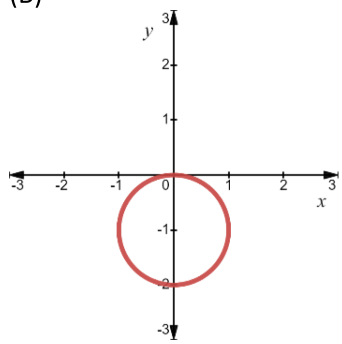
(A) 0.707 (B) 1.414 (C) 2.828 (D) 7.069

(12) Which of the following represents the graph of the polar curve $r = -2\sec\theta$?

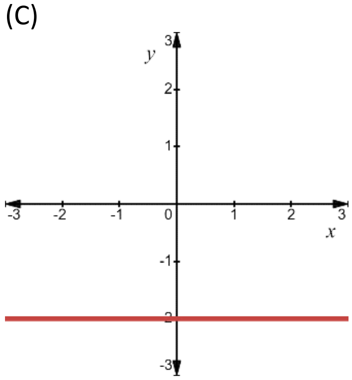
(A)



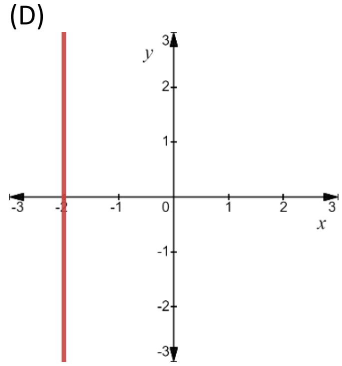
(B)



(C)



(D)



(13) Let $r = f(\theta)$ be the polar curve whose equation is given by $r = -2\theta^3$. What is the slope of the line tangent to $f(\theta)$ at the point where $\theta = \frac{\pi}{2}$?

(A) $-\frac{6}{\pi}$

(B) $\frac{6}{\pi}$

(C) $-\frac{3\pi^2}{8}$

(D) $\frac{3\pi^2}{8}$

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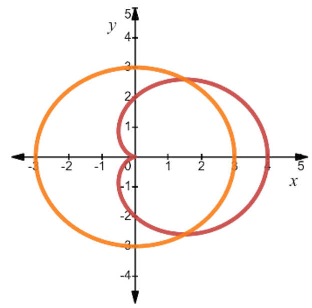
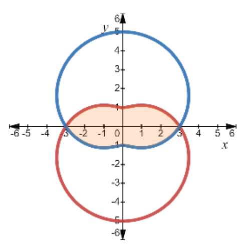
(14) The figure below shows the graphs of the polar curves $r = 3 \; and \; r = 2 + 2\cos\theta$. Find the area of the region inside $r = 2 + 2\cos\theta$ and outside $r = 3$.



(A) 9.425 (B) 4.712 (C) 4.653 (D) 2.326

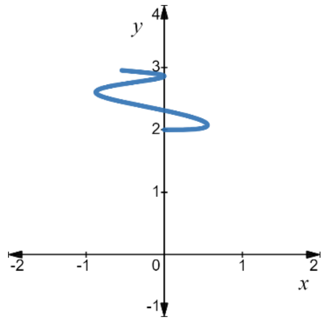
(15) The polar curves $r = 3 - 2\sin\theta$ and $r = 3 + 2\sin\theta$ are shown in the figure below. Which of the following expressions gives the total area of the shaded regions?



(A) 5.279 (B) 10.558 (C) 17.279 (D) 34.558

Part 2 Free Response- Calculator use is permitted.

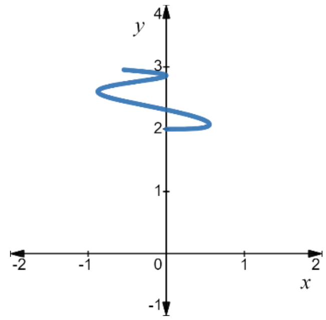
(1)



An ant is crawling on a wall in such a way that its position at time t seconds can be represented by $(x(t), y(t))$ in the xy-plane (graphed above). Both x and y are measured in centimeters, and t is measured in seconds. The ant's initial position at time t = 0 is (0, 2) and the ant crawls for 3 seconds on the wall. For $0 \leq t \leq 3$, $\frac{dx}{dt} = 2\cos(t^2 + 1)$ and $\frac{dy}{dt} = 3\sin\left(\frac{t^3}{e^{2t}}\right)$.

(a) Find the speed of the ant at time t = 2 seconds. Show the work that leads to your answer.

(b) Find the total distance the ant travels from t = 0 to time t = 3 seconds. Show the setup for your calculations.

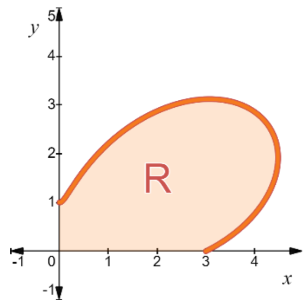


An ant is crawling on a wall in such a way that its position at time t seconds can be represented by $(x(t), y(t))$ in the xy-plane (graphed above). Both x and y are measured in centimeters, and t is measured in seconds. The ant's initial position at time t = 0 is (0, 2) and the ant crawls for 3 seconds on the wall. For $0 \leq t \leq 3$, $\frac{dx}{dt} = 2\cos(t^2 + 1)$ and $\frac{dy}{dt} = 3\sin\left(\frac{t^3}{e^{2t}}\right)$.

(c) Find the ant's position at time t = 1.5 seconds. Show the setup for your calculations.

(d) Find the time t, $0 \leq t \leq 3$, when the ant is farthest to the left. Justify your answer.

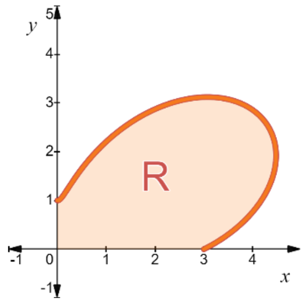
(2) Consider the polar curve defined by the equation $r = 3 + 2\sin(3\theta)$ on $0 \le \theta \le \frac{\pi}{2}$. Let Region R be the space in Quadrant I enclosed by the polar curve and the lines $\theta = 0$ and $\theta = \frac{\pi}{2}$.



(a) Find the area of R. Show the setup that leads to your calculations.

(b) Find the equation of the line that is tangent to the graph of $r = 3 + 2\sin(3\theta)$ at the point where $\theta = \frac{\pi}{3}$. Show the setup for your calculations.

Consider the polar curve defined by the equation $r = 3 + 2\sin(3\theta)$ on $0 \leq \theta \leq \frac{\pi}{2}$. Let Region R be the space in Quadrant I enclosed by the polar curve and the lines $\theta = 0$ and $\theta = \frac{\pi}{2}$.



(c) Consider the straight line $y = mx$. Write, but do not solve, an equation involving m such that the line divides the section of the graph of $r = 3 + 2\sin(3\theta)$ in the first quadrant into two regions of equal area.