Name Answer Key

Calc Test Review – Related Rates

Part I - Implicitly Differentiate:

1) **With respect to x: $x^3y - 3y + 4x = 8$**

$x^3\frac{dy}{dx} + 3x^2y - 3\frac{dy}{dx} + 4 = 0$

$x^3\frac{dy}{dx} - 3\frac{dy}{dx} = -3x^2y - 4$

$\frac{dy}{dx}(x^3 - 3) = -3x^2y - 4$

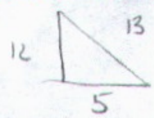
$\frac{dy}{dx} = \frac{-3x^2y-4}{(x^3-3)}$ or $\frac{dy}{dx} = \frac{3x^2y+4}{3-x^3}$

2) **With respect to t: $4ab + 4c - 3d = 5c^2d$**

$4a\frac{db}{dt} + 4b\frac{da}{dt} + 4\frac{dc}{dt} - 3\frac{dd}{dt} = 5c^2\frac{dd}{dt} + 10cd\frac{dc}{dt}$

Part II – Solve the Related Rates:

3) **A 13 meter ladder rests 12 meters high on a wall. In an instant, the ladder slips down the wall at 9.8 meters per second. How fast is the base of the ladder moving away from the wall at this instant?**



$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

length of ladder does not change

$2(5)\left(\frac{da}{dt}\right) + 2(12)(-9.8) = 2(13)(0)$

$10\frac{da}{dt} + 235.2 = 0$ $\frac{da}{dt} = \frac{-235}{10}$

$\therefore \frac{da}{dt} = 23.52$ m/s away from wall

4) **A bottle of cooking oil punctures inside a cabinet. The oil spills out slowly in a circle. At the instant the radius of the spill is 4cm, the area of the spill is increasing at a rate of 5cm per minute.**

a. **How quickly is the radius of the spill increasing?**

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$5 = 2\pi(4)\frac{dr}{dt}$

$5 = 8\pi\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{8\pi}$ or .198 cm/minute

b. **If the cooking oil bottle is 20 cm from the edge of the cabinet, how long until the oil spills over the edge?**

$\frac{20 \text{ cm}}{.198 \text{ cm/minute}}$ = 101 minutes or 1hr. 41 minutes

5) **You are closing your pool for the winter. The pool is cylindrical in shape with a circumference of 28π feet. How fast is the volume of pool water being emptied from the pool at the instant that the water level of the pool is 3 feet high** and the height is decreasing at a rate of .15 feet per hour?

$V = \pi r^2h$

$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi r\frac{dr}{dt}h$

the radius of the pool is not changing

$\frac{dV}{dt} = \pi(14)^2(-.15) + 2\pi(14)(0)(3)$

$\frac{dV}{dt} = -92.36$ cubic feet per hour

6) **The height of a cone is always 8 times its radius. If the volume of a cone is increasing at a rate of 10 feet per minute, how fast is the radius increasing when the radius is 2 feet?**

need $\frac{dr}{dt}$

$V = \frac{1}{3}\pi r^2h$

$h = 8r$

$V = \frac{1}{3}\pi r^2 8r$

$V = \frac{8}{3}\pi r^3$

$\frac{dV}{dt} = 8\pi r^2\frac{dr}{dt}$

$10 = 8\pi(2)^2\frac{dr}{dt}$

$10 = 32\pi\frac{dr}{dt}$

$\frac{10}{32\pi} = \frac{dr}{dt}$

or .0995 ft/minute